Addendum to

Equilibria under Deferred Acceptance: Dropping Strategies, Filled Positions, and Welfare

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We refer to Jaramillo et al. (2014) for the model and the notation. Example 3 in Jaramillo et al. (2014), which is reproduced below, exhibits a many–to–one market for which there exists an equilibrium outcome such that simultaneously for each side of the market there is an agent that is strictly worse off and another agent that is strictly better off than in any stable matching.

After the publication of Jaramillo et al. (2014), we have employed exhaustive computer calculations1 to find the full set of matchings that are obtained in Nash equilibria. Below we report on our findings.

Example 1. [Jaramillo et al., 2014, Example 3]
Consider a many–to–one market \((P_S, \succ_H)\) with 6 students, 3 hospitals, and preferences over individual partners \(P\) given by the columns in Table 1. The hospitals have quota 2. Moreover, assume that \(\{s_1, s_4\} \succ_{h_2} \{s_5, s_6\}\). Note that this assumption does not contradict the responsiveness of \(h_2\)’s preferences.

What we knew: One easily verifies that the student-optimal stable matching \(\mu := \varphi^S(P)\) is given by

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1The programs implemented in Matlab are available upon request.
Table 1: Preferences $P$ in Example 1

<table>
<thead>
<tr>
<th>Students</th>
<th></th>
<th>Hospitals</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s_1$</td>
<td>$s_2$</td>
<td>$s_3$</td>
</tr>
<tr>
<td>$h_2$</td>
<td>$h_1^*$</td>
<td>$h_1$</td>
</tr>
<tr>
<td>$h_1^*$</td>
<td>$h_3^*$</td>
<td>$h_2$</td>
</tr>
</tbody>
</table>

which is the boxed matching in Table 1. The only other stable matching in $\Sigma(P)$ is given by

<table>
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<th></th>
<th>Hospitals</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s_1$</td>
<td>$s_2$</td>
<td>$s_3$</td>
</tr>
<tr>
<td>$h_1^*$</td>
<td>$h_1^*$</td>
<td>$h_3$</td>
</tr>
<tr>
<td>$h_2^*$</td>
<td>$h_3^*$</td>
<td>$h_2$</td>
</tr>
<tr>
<td>$s_3$</td>
<td>$s_6$</td>
<td>$s_4^*$</td>
</tr>
</tbody>
</table>

which is the matching marked with * in Table 1. The difference between $\mu$ and $\mu^*$ is in the matches of students $s_4$ and $s_6$.

Consider the dropping strategies $Q'_{h_1} = s_2, s_3$, $Q'_{h_2} = s_4, s_1$, and $Q'_{h_3} = s_5, s_6, s_4$ for hospitals $h_1$, $h_2$, and $h_3$. It is easy to verify that $Q' = (Q'_{h_1}, Q'_{h_2}, Q'_{h_3})$ is an equilibrium and that it induces the matching

$$
\mu' := \phi^S(Q') : \begin{vmatrix}
    h_1 & h_2 & h_3 \\
    \{s_2, s_3\} & \{s_1, s_4\} & \{s_5, s_6\}
\end{vmatrix}
$$

which is the boldfaced matching in Table 1. At the unstable equilibrium outcome $\mu'$, for each side of the market there is an agent that is strictly worse off than in any stable matching and there is another agent that is strictly better off than in any stable matching. Indeed, for the hospitals' side we observe that $\mu(h_1) = \mu^*(h_1) \succ_h \mu'(h_1)$, but $\mu'(h_3) \succ_h \mu^*(h_3) \succ_h \mu(h_3)$. And similarly, for the students' side we find that $\mu(s_5) = \mu^*(s_5) P_{s_5} \mu'(s_5)$, but $\mu'(s_1) P_{s_1} \mu(s_1) = \mu^*(s_1)$.

What we now know as well: Consider the dropping strategies $Q''_{h_1} = s_1, s_2$, $Q''_{h_2} = s_4, s_6$, and $Q''_{h_3} = s_5, s_3, s_4$ for hospitals $h_1$, $h_2$, and $h_3$. It is easy to verify that $Q'' = (Q''_{h_1}, Q''_{h_2}, Q''_{h_3})$ is an equilibrium and that it induces the matching

$$
\mu'' := \phi^S(Q'') : \begin{vmatrix}
    h_1 & h_2 & h_3 \\
    \{s_1, s_2\} & \{s_4, s_6\} & \{s_3, s_5\}
\end{vmatrix}
$$
which is the underlined matching in Table 1. Matching $\mu''$ does not exhibit the same welfare features as $\mu'$, i.e., it is not true that at the unstable equilibrium outcome $\mu''$, for each side of the market there is an agent that is strictly worse off than in any stable matching and there is another agent that is strictly better off than in any stable matching. (To see this note that at $\mu''$ there is no student that is strictly better off than at any stable matching. Note also that at $\mu''$ there is no school that is strictly worse off than at any stable matching.)

Exhaustive computer calculations show that there is no other matching that can be sustained at a Nash equilibrium. Hence, $O(\succ_H) = \{\mu, \mu^*, \mu', \mu''\}$. Since $Q'$ and $Q''$ consist of dropping strategies and since $\mu$ and $\mu^*$ can be obtained in some equilibria that consist of dropping strategies (Jaramillo et al., 2014, Proposition 1), all equilibrium outcomes can be obtained in equilibria that consist of dropping strategies.

References